Technical Memorandum 221201

DYNAMIC RESPONSE OF A DRIVE WITH DAMPING AND AN IMBEDDED TORQUEMETER

Application Note 221101D describes the response of installed Torquemeters in drives with and without damping. Equations are presented for the undamped case but omitted for the damped case. This memo describes an efficient, computer based method for analyzing the damped case.

Figure 1 is a schematic of the drive discussed in Application Note 221101D. By way of reminder a motor with inertia $\mathbf{J_m}$ drives a pump with inertia $\mathbf{J_p}$ through a Torquemeter with stiffness **k**. The motor inertia sees damping Ω_m and the pump inertia sees damping Ω_p.

As discussed in the Application Note, in the real world all shaft networks see a constant or average torque drive with superimposed oscillatory components. The constant portion of the torque is balanced by the constant load and bearing friction. Additionally, there are dynamic variations of inertia torques and the fluctuations in viscous friction caused by drive speed variations.

The traditional solution is to write the differential equations of motion describing the network, solve them, substitute numeric values for the network parameters, calculate the performance and plot the results. That procedure is by no means trivial; even the final calculations are complex and subject to error.

At Himmelstein, we use a combined analog/digital method. First the mechanical network is converted to an electric network using the electric analogs listed below. That conversion is valid because the differential equations governing the response of the electric and mechanical elements are identical. For example:

Inertia Torque is $J\frac{dw}{dt}$ where $\frac{dw}{dt}$ is angular acceleration; the derivative of angular velocity

Electric Current is $C\frac{dv}{dt}$ where $\frac{dv}{dt}$ is the derivative of voltage

Thus, Current is the analog of Torque, Capacitance is the analog of Inertia, Voltage is the analog of Angular Velocity .

The table below lists all related analogies. It should be mentioned that other valid analogies are possible but, we use these for rotating torque applications.

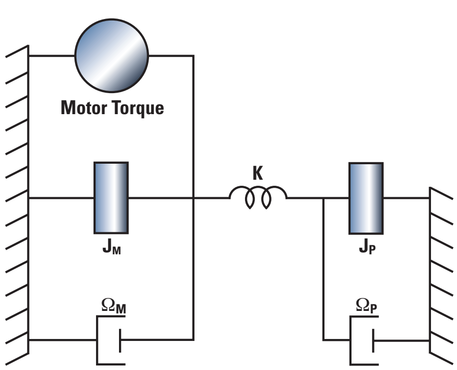

Figure 1. Driveline Mechanical Schematic

Inertia/Capacitance Analogy

Inertia (J)	Capacitance (C)
Torque (T)	Current (I)
Velocity (ω)	Voltage (V)
Damping (Ω)	Conductance ($\frac{1}{R}$)
Compliance ($\frac{1}{k}$)	Inductance (L)
Displacement (θ)	$\int_0^t v\,dt$

Drive Component	Mechanical value	Electric Analog Value
Motor Inertia (J_m)	100 in-lbf s^2	100 farad (C_m)
Pump Inertia (J_p)	172 in-lbf s^2	172 farad (C_p)
Torquemeter Stiffness (k)	309,000,000 lbf-in/rad	3.2364E-09 henry (L_k)
Motor Damping (Ω_m)	0	10^{+15} ohm (R_m)
Pump Damping (Ω_p)	0	10^{+15} ohm (R_p)

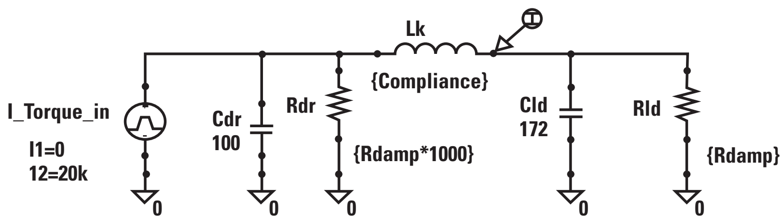

Figure 2. Electric Analog of Driveline

Response of Motor-Pump Driveline With MCRT 87007V(25-3) Installed

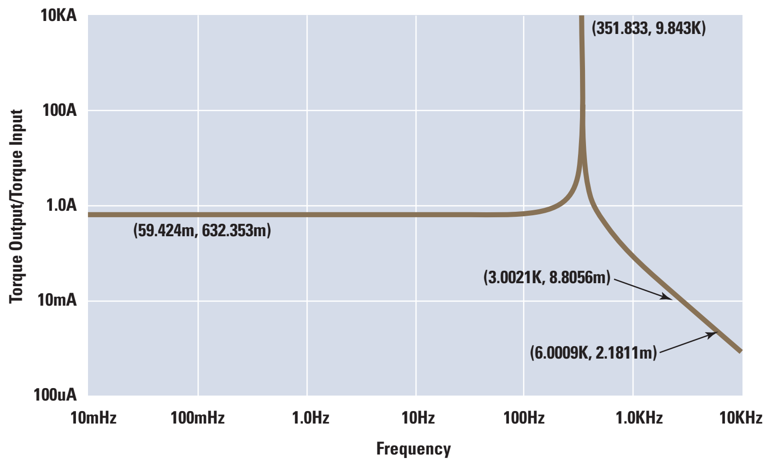

Figure 3. Computer Solution With Damping Set to Zero - Rm = Rp = 10^{+15} ohms ≅ Infinity.

Response to Step Input With Damping Varied and MCRT 87007V(25-3) Installed

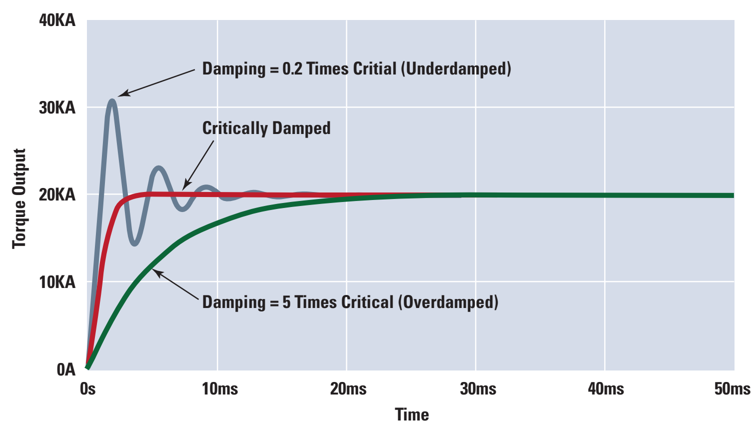

Figure 4. Response to Step Input With MCRT 87007V(25-3) Torquemeter Installed.

Figure 1 is the system mechanical network. **Figure 2** is its electric analog. Since the simulation program used doesn't handle *conductance, damping* is entered as a resistor value that is the reciprocal of damping. For example, to convert the analog of the shaft network to the undamped case, **Rm** and **Rp** must become infinite. As a practical matter, enter **10^{+15}** ohms for **Rm** and **Rp**. Because the reciprocal of **10^{+15}** is essentially zero, damping is essentially zero, i.e., **Rm** and **Rp** are effectively infinite and, therefore, removed from the electric analog.

The output Torque is the current through the Inductor (spring) **L_k**. To complete the entry, use the following values to simulate the undamped case with the MCRT 87007V(25-3) Torquemeter installed.

Figure 3 is a printout from the computer simulation of this undamped network with the MCRT Torquemeter installed. It is identical to the result obtained with equation 1 of Application Note 221101D; at low frequencies the gain is 0.632, the natural frequency is 351.8 Hz and the attenuation at 3 and 6 kHz match those obtained from Equation 1.

Equation 1 of the Application Note is the classic response for a spring coupled inertia pair without damping. The fact the two results are identical validates the use of the electric analogy and the correctness of both the mechanical network and its electric analog.

Although there are many simulation

programs suitable for this application, we use Pspice. It is robust, has high resolution, performs parametric solutions, has powerful display and plotting capabilities, and performs both transient and steady state analyses.

Figure 4 is a printout from the computer solution of the damped case with a torque step input and the MCRT Torquemeter installed. It should be mentioned that the damped analysis was made with virtually all of the damping in the pump. That is consistent with the assumption that the torque input is the motor developed torque, and as a result, virtually all the developed torque is delivered to the pump and virtually all the mechanical power is consumed by the pump.

The division of network damping, the damping ratio, will have a small effect on system response. However, regardless of the damping ratio, the damped system bandwidth is always lower than the un-damped case. **Figure 5** shows the effect of damping ratio on Frequency Response and Torque Division. Damping has been adjusted to critical because critical damping provides the widest bandwidth and fastest transient response. As shown, the damping ratio has a relatively minor effect on system response.

Induction motors are often used to drive pumps. Both single and three phase induction motors produce torque ripple at twice the line frequency. If all three line phases have identical amplitudes (and are precisely 120 degrees apart), and all motor windings have identical impedances independent of angular position, then a three phase induction motor won't have ripple. Since those requirements are never met, real three phase motors produce torque ripple at twice the line frequency.

To optimize efficiency, the motors are frequently powered by variable frequency

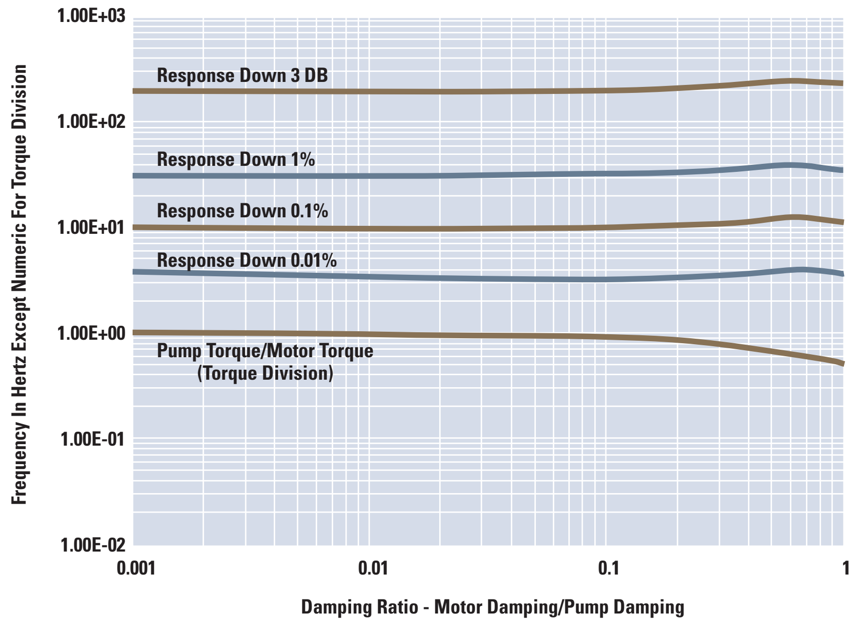

Figure 5. Effect of Damping Ratio on Frequency Response and Torque Division.

Frequency Response of Competitive 3kNm Device With Low Damping

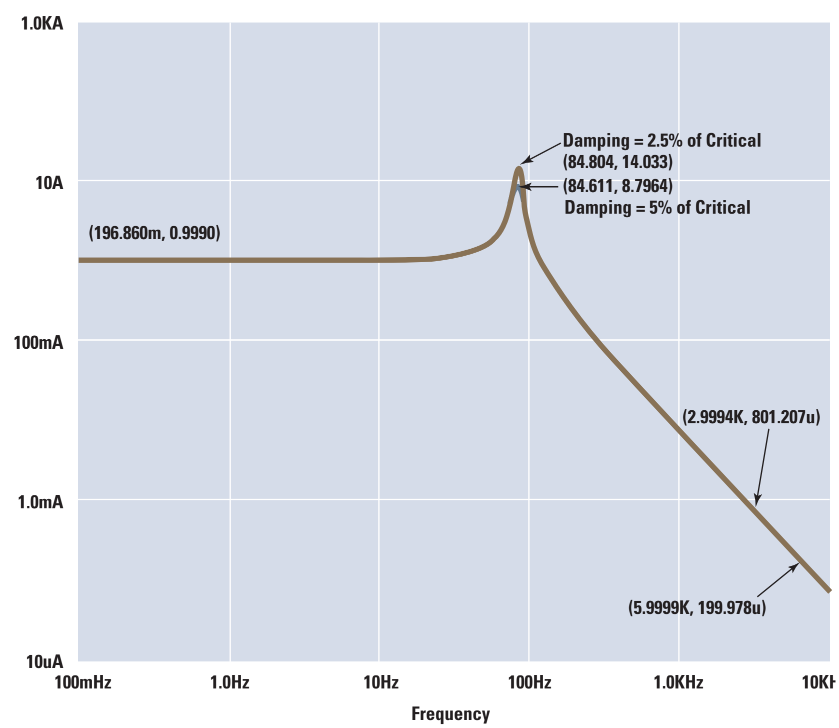

Figure 6. Frequency Response of Competitive 3kNm Torquemeter With Low Damping.

Damping = 2.5 and 5.0% of Critical

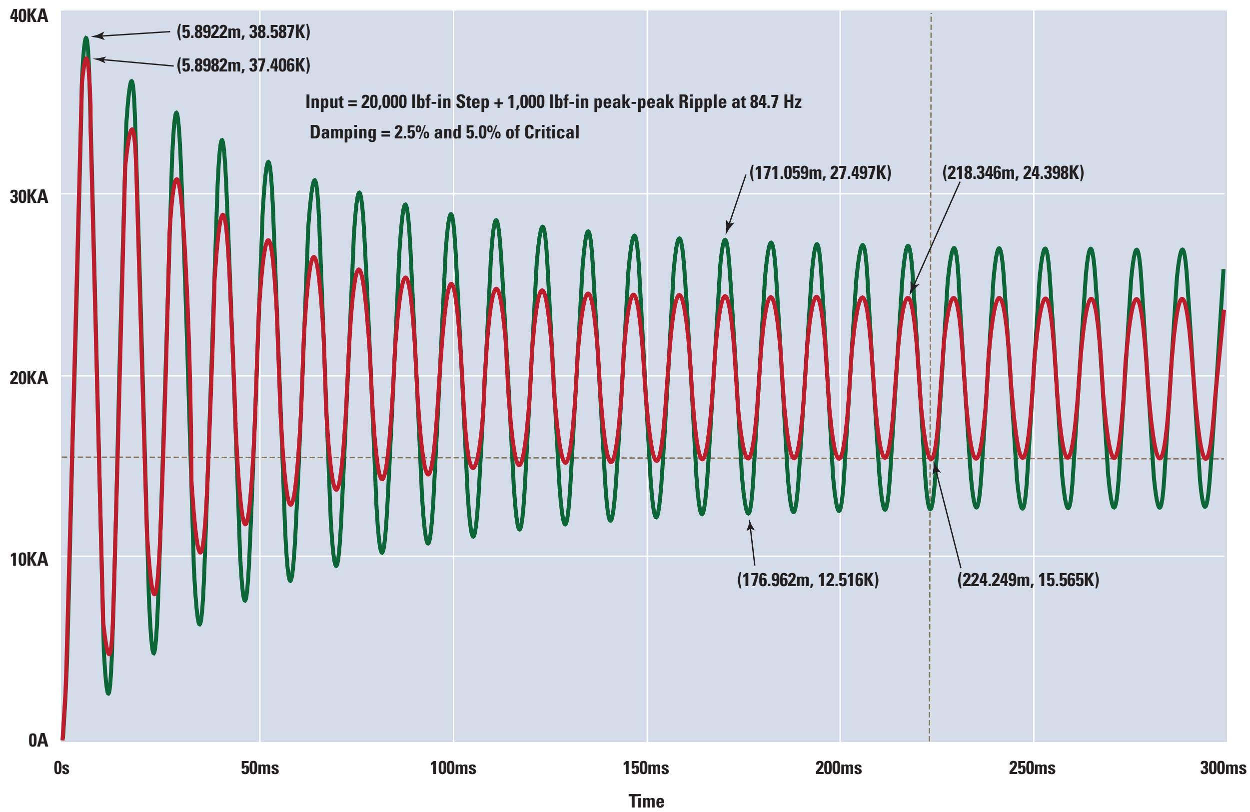

Figure 7. Competitive 3 kNm Device Response to Step + Ripple at Drive Natural Frequency

drives (VFD's). Thus, it's likely the ripple torque frequency will coincide with the network natural frequency at some speed in the operating range. That can produce a significant torque multiplication which could result in fatigue failure or unsafe stress levels. The risk is increased at startup, other transient load conditions and, when the Torquemeter is compliant.

To illustrate, we ran a frequency response with a competitive 3 kNm Torquemeter installed and damping set at 2.5 and 5.0 percent of critical; see **Figure 6.** The network natural frequency is about 84.7 Hz., torque multiplication is 8.8 and 14 times at resonance and signal attenuation is 1,249 and 4,997 times at 3 and 6 kHz.

Next we ran a transient response for the same setup using a step input of 20,000 lbf-in with a 1,000 lbf-in peak-peak, 84.7 Hz. ripple component; see **Figure 7.** The peak torques exceed the allowable Torquemeter Overrange and are perilously close to the allowable Overload. The ripple torque sees an 8.8 to 15 times multiplication, depending on damping. There is a risk of fatigue failure if operated under these conditions for long periods. That risk increases with higher ripple amplitudes. Using a stiffer Torquemeter increases the network natural frequency and, therefore, reduces the likelihood it will equal twice the drive frequency.

The following references contain additional information on this subject.

1. S. Himmelstein and Company Publication: *Technical Memorandum 8150, Avoiding the Destructive Effects of Torsional Resonance.*

2. Harris, C. M., and D. M. Crede, *Shock and Vibration Handbook* (3 Volumes), McGraw Hill Book Company, 1961

3. Karplus, W. C., and W. W. Soroka, *Analog Methods*, McGraw Hill Book Company, 1959

4. Gardner, M. F., and J. L. Barnes, *Transients in Linear Systems*, John Wiley & Sons, Inc.,1942

5. Tuinenga, P. W., *Spice - A Guide to Circuit Simulation & Analysis Using Pspice*, Prentice Hall, 1988

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